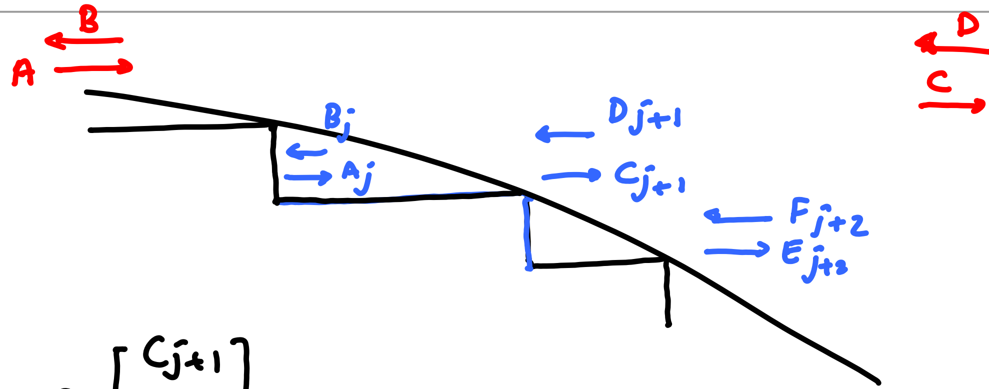
TMM

Note Title 2/6/2008



$$\begin{bmatrix} A_j \\ B_j \end{bmatrix} = P_j \begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix}$$

$$\begin{bmatrix} C_{j+1} \\ D_{j+1} \end{bmatrix} = P_{j+1} \begin{bmatrix} F_{j+2} \\ E_{j+2} \end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \underbrace{P_1 P_2 \cdots P_N}\begin{bmatrix} C \\ D \end{bmatrix} = \underline{P}\begin{bmatrix} C \\ D \end{bmatrix}$$

$$\underline{P} = \prod_{j=1}^{N} P_j$$

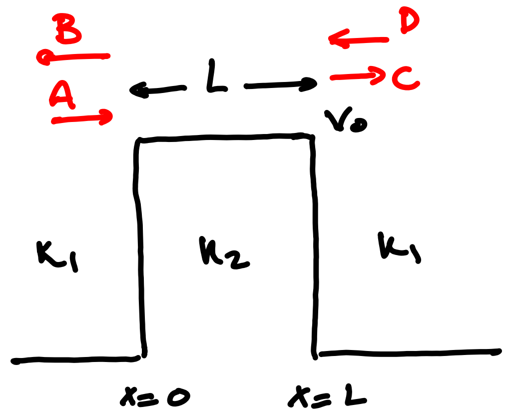
Since the particle is introduced from the left $A = 1$

and if there is no reflection after the last step $D = 0 \Rightarrow$

$$\begin{bmatrix} 1 \\ B \end{bmatrix} = \underline{P}\begin{bmatrix} C \\ 0 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} C \\ 0 \end{bmatrix} \Rightarrow \begin{cases} 1 = P_{11} C \\ B = P_{21} C \end{cases}$$

$$\text{Transmission probability} = \left|\frac{C}{A}\right|^2 = |C|^2 = \left|\frac{1}{P_{11}}\right|^2$$

Example Transmission probability for a rectangular potential barrier



$$\begin{cases} \psi_1 = A\, e^{ik_1x} + B\, e^{-ik_1x} \\ \psi_2 = C\, e^{ik_2x} + D\, e^{-ik_2x} \end{cases}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = P_{1\,step}\, P_{1\,free}\, P_{2\,step} \begin{bmatrix} C \\ D \end{bmatrix}$$

$$= \frac{1}{2}\frac{1}{2}\begin{bmatrix} 1+\frac{k_2}{k_1} & 1-\frac{k_2}{k_1} \\ 1-\frac{k_2}{k_1} & 1+\frac{k_2}{k_1} \end{bmatrix}\begin{bmatrix} e^{-ik_2L} & 0 \\ 0 & e^{ik_2L} \end{bmatrix}\begin{bmatrix} 1+\frac{k_1}{k_2} & 1-\frac{k_1}{k_2} \\ 1-\frac{k_1}{k_2} & 1+\frac{k_1}{k_2} \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} (1+\frac{k_2}{k_1})e^{-ik_2L} & (1-\frac{k_2}{k_1})e^{ik_2L} \\ (1-\frac{k_2}{k_1})e^{-ik_2L} & (1+\frac{k_2}{k_1})e^{ik_2L} \end{bmatrix}\begin{bmatrix} 1+\frac{k_1}{k_2} & 1-\frac{k_1}{k_2} \\ 1-\frac{k_1}{k_2} & 1+\frac{k_1}{k_2} \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} (1+\frac{k_2}{k_1})(1+\frac{k_1}{k_2})e^{-ik_2L} + (1-\frac{k_2}{k_1})(1-\frac{k_1}{k_2})e^{ik_2L} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix}$$

$$P_{11} = \frac{1}{4k_1k_2}\left[(k_1+k_2)^2 e^{-ik_2L} - (k_1-k_2)^2 e^{ik_2L}\right]$$

$$= \frac{1}{4k_1k_2}\left[(k_1^2+k_2^2)(e^{-ik_2L} - e^{ik_2L}) + 2k_1k_2(e^{ik_2L} + e^{-ik_2L})\right]$$

Transmission when $E \geq V_0$

$$P_{11} = \frac{1}{4k_1k_2}\left[(k_1^2+k_2^2)(-2i\sin k_2L) + 2k_1k_2(2\cos k_2L)\right]$$

$$= -i\,\frac{k_1^2+k_2^2}{2k_1k_2}\sin k_2L + \cos k_2L$$

$$\text{Trans} = \frac{1}{|P_{11}|^2} = \frac{1}{P_{11}P_{11}^*} = \left[\left(\frac{k_1^2+k_2^2}{2k_1k_2}\right)^2 \sin^2 k_2L + \underbrace{\cos^2 k_2L}_{1-\sin^2 k_2L}\right]^{-1}$$

$$= \left[\left(\left(\frac{k_1^2+k_2^2}{2k_1k_2}\right)^2 - 1\right)\sin^2 k_2L + 1\right]^{-1}$$

$$= \left[\left(\frac{\overbrace{k_1^4+k_2^4+2k_1^2k_2^2-4k_1^2k_2^2}^{(k_1^2-k_2^2)^2}}{4k_1^2k_2^2}\right)\sin^2 k_2L + 1\right]^{-1}$$

$$\boxed{\text{Trans}(E \geq V_0) = \frac{1}{1+\left(\frac{k_1^2-k_2^2}{2k_1k_2}\right)^2 \sin^2 k_2L}}$$

So when $\sin^2 k_2L = 0 \Rightarrow \text{Trans} = 1$

$\Rightarrow k_2L = n\pi \quad n = 1, 2, 3, \ldots$ <u>resonance</u>

when $E < V_0$

k_2 becomes imaginary : $k_2 \to ik_2$

where $\underbrace{E - V_0}_{\ominus} = \dfrac{\hbar^2 k_2^2}{2m}$

$$\text{Trans}\,(E<V_0) = \frac{1}{1 + \left(\frac{k_1^2 - (ik_2)^2}{2k_1(ik_2)}\right)^2 \underbrace{\sin^2 ik_2L}_{\sinh^2 k_2 L}}$$

$$\boxed{\text{Trans}\,(E<V_0) = \frac{1}{1 + \left(\frac{k_1^2 + k_2^2}{2k_1k_2}\right)^2 \sinh^2(k_2L)}}$$

Transmission is through tunneling in this case